Name _____

MATH 350: Graph Theory and Combinatorics. Fall 2013 - Midterm Exam

Thursday, October 10th, 2013, 16:35-17:25

The questions have to be answered in the booklets provided.

You can choose which two questions to answer. Indicate your choice on the front page. Only the two chosen questions will be graded.

Write your answers clearly. Justify all your answers.

You can consult your notes and textbooks. Use of calculators, computers, cell-phones, etc. is not permitted.

Problem	Your choice	Your score
1		
2		
3		
Total		

1. Let G be a simple graph such that $|V(G)| = 8$ and G has three vertices with degree 3, two vertices with degree 4 and three vertices with degree 5.

a) Is G necessarily connected?

b) Can G be bipartite?

c) Does G necessarily contain a Hamiltonian cycle?

2. Let G be a connected graph with m edges $e_1, e_2, \ldots, e_m$.
Let $w : E(G) \to \mathbb{R}_+$ be such that $w(e_i) = 10^i$ for $i = 1, 2, \ldots, m$.
Let T be the min-cost tree of G with respect to w.

Prove that $\text{dist}_T(u, v) = \text{dist}_G(u, v)$ for every pair of vertices $u, v \in V(G)$, where the distances are measured with respect to w.

3. Let $k \geq 0$ be an integer and let G be a simple bipartite graph with bipartition (A, B). Suppose that every vertex of G has degree $\geq k$ and that there exist $a \in A, b \in B$ such that $\deg(a) = \deg(b) = k + 1$.

Prove that G contains a matching of size $k + 1$.