Name _____

MATH 350: Graph Theory and Combinatorics. Fall 2014 - Midterm Exam

Wednesday, October 15th, 2014, 10:35-11:25

The questions have to be answered in the booklets provided.

You can choose which two questions to answer. Indicate your choice on the front page. Only the two chosen questions will be graded.

Write your answers clearly. Justify all your answers.

You can consult your notes and textbooks. Use of calculators, computers, cell-phones, etc. is not permitted.

Problem	Your choice	Your score
1		
2		
3		
Total		

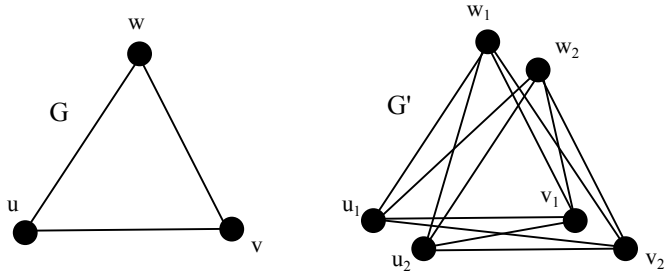
1. Let G be a simple graph such that $|V(G)| = 8$ and G has four vertices with degree 3 and four vertices with degree 5.

a) Is G necessarily connected?

b) Can G be bipartite?

c) Does G necessarily contain a path of length 6?

2. Let G be a loopless graph in which the degree of every vertex is even. Show that the line graph $L(G)$ of G contains a matching of size $\geq |E(G)|/3$.



3. Let G be a k-connected graph. A graph G' is obtained from G by replacing every vertex $v \in V(G)$ by a pair of vertices v_1 and v_2, such that if $u, v \in V(G)$ are adjacent in G then u_1 and u_2 are adjacent to both v_1 and v_2 in G'. (See figure for the case $G = K_3$.) Show that G' is $2k$-connected.