Ramsey Classes by Partite Construction I

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Joint work with Jaroslav Nešetřil

Permutation Groups and Transformation Semigroups 2015

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We consider relational structures in language *L* without function symbols.

Definition

A class C (of finite relational structures) is Ramsey iff

$$\forall_{\mathbf{A},\mathbf{B}\in\mathcal{C}}\exists_{\mathbf{C}\in\mathcal{C}}:\mathbf{C}\longrightarrow(\mathbf{B})_{2}^{\mathbf{A}}.$$

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 $\begin{pmatrix} B \\ A \end{pmatrix}$ is set of all substructures of **B** isomorphic to **A**. $\mathbf{C} \longrightarrow (\mathbf{B})_2^{\mathbf{A}}$: For every 2-coloring of $\begin{pmatrix} C \\ A \end{pmatrix}$ there exists $\widetilde{\mathbf{B}} \in \begin{pmatrix} C \\ B \end{pmatrix}$ such that $\begin{pmatrix} \widetilde{\mathbf{A}} \\ \widetilde{\mathbf{A}} \end{pmatrix}$ is monochromatic.

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Example (Non-example)

The class of all directed graphs is not Ramsey.



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Example (Non-example)

The class of all directed graphs is not Ramsey.



Given **C** consider arbitrary linear order. Color edges red if they go forward in the linear order and blue otherwise.

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Ramsey classes are amalgamation classes



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Ramsey classes are amalgamation classes



Nešetřil, 1989: Under mild assumptions Ramsey classes have amalgamation property.



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Classification programme

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Ramsey classes	\implies	amalgamation classes
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lifts of homogeneous	$\Leftarrow\!\!=$	homogeneous structures

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Many amalgamation classes are given by the classification programme of homogeneous structures.

Can we always find a Ramsey lift?

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Theorem (Nešetřil, 1989)

All homogeneous graphs have Ramsey lift.

Theorem (Jasiński,Laflamme,Nguyen Van Thé,Woodrow, 2014) All homogeneous digraphs have Ramsey lift.

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Map of Ramsey Classes



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interval graphs permutations cyclic orders unions of complete graphs linear orders restricted

A structure **A** is called complete (or irreducible) if every pair of distinct vertices belong to a relation of **A**.

 $\operatorname{Forb}_{\mathcal{E}}(\mathcal{E})$ is a class of all finite structures **A** such that there is no embedding from $\mathbf{E} \in \mathcal{E}$ to **A**.

Theorem (Nešetřil-Rödl Theorem, 1977)

- Let L be a finite relational language.
- Let *E* be a set of complete ordered L-structures.
- The then class $\operatorname{Forb}_{E}(\mathcal{E})$ is a Ramsey class.

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Explicitly: For every $\mathbf{A}, \mathbf{B} \in \operatorname{Forb}_{\mathcal{E}}(\mathcal{E})$ there is $\mathbf{C} \in \operatorname{Forb}_{\mathcal{E}}(\mathcal{E})$ such that $\mathbf{C} \longrightarrow (\mathbf{B})_2^{\mathbf{A}}$.

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Examples of Ramsey lifts

Example

Graphs with order are Ramsey.



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Examples of Ramsey lifts



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Examples of Ramsey lifts



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permutations cyclic orders unions of complete graphs	acyclic graphs partial orders <i>Kn</i> -free graphs
linear orders	graphs
restricted	free

interval graphs permutations cyclic orders unions of complete g linear orders



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Nešetřil-Rödl: The Partite Construction and Ramsey Set Systems (1989)

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Definition (A-partite system)

Let **A** be an ordered relational structure on vertices $\{1, 2, \dots a\}$.

An **A**-partite system is a tuple $(\mathbf{A}, \mathcal{X}_{\mathbf{B}}, \mathbf{B})$ where **B** is structure and $\mathcal{X}_{\mathbf{B}} = \{X_{\mathbf{B}}^1, X_{\mathbf{B}}^2, \dots, X_{\mathbf{B}}^a\}$ partitions vertex set of **B** into *a* classes $(X_{\mathbf{B}}^i \text{ are called } parts \text{ of } \mathbf{B})$ such that:

- ordering satisfies $X_{\mathbf{B}}^1 < X_{\mathbf{B}}^2 < \ldots < X_{\mathbf{B}}^a$;
- 2 mapping (projection) π which maps every $x \in X_{\mathsf{B}}^{i}$ to i (i = 1, 2, ..., a) is a homomorphism;
- every tuple in every relation of **B** meets every class $X_{\mathbf{B}}^{i}$ in at most one element.

$$\mathbf{A} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} X_{\mathbf{B}}^{1} & \bullet & \bullet \\ X_{\mathbf{B}}^{2} & \bullet \\ X_{\mathbf{B}^{2} & \bullet \\ X_{\mathbf{B}}^{2} & \bullet \\ X_{\mathbf{B}^{2} & \bullet \\ X_{\mathbf{B}}^{$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{B} \\ \mathbf{C} \end{bmatrix} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C} \end{bmatrix} \mathbf{C} = \mathbf{C}$$

Construction outline:

• Put n such

$$n \longrightarrow (|\mathbf{B}|)_2^{|\mathbf{A}|}.$$

(For every coloring of |A| tuples in $\{1, 2, ..., n\}$ there exists monochromatic subset of size $|\mathbf{B}|$). Here n = 6.

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Picture 0: |K|_n-partite system P₀ s.t. for every coloring of copies of A in P₀ where the color of a copy A depends only on a projection π(A) there exists a monochromatic copy of B.

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- Enumerate by $\mathbf{A}_1, \dots, \mathbf{A}_N$ all possible projections of copies of \mathbf{A} in \mathbf{P}_0 .

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- Pictures 1...n: K_n-partite systems P₁,...P_N s.t. for every coloring of copies of A in P_i there exists a copy of P_{i-1} where all copies of A with projection A_i are monochromatic.

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Picture 0: K_n -partite system P_0 s.t. for every coloring of copies of A in P_0 where the color of a copy \widetilde{A} depends only on a projection $\pi(\widetilde{A})$ there exists a monochromatic copy of **B**.



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Picture 2: K_n -partite system P_2 s.t. for every coloring of copies of **A** in P_2 there exists a copy of P_1 where all copies of **A** with projection A_2 are monochromatic.

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Picture 2: K_n -partite system P_2 s.t. for every coloring of copies of **A** in P_2 there exists a copy of P_1 where all copies of **A** with projection A_2 are monochromatic.



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Picture 2: K_n -partite system P_2 s.t. for every coloring of copies of **A** in P_2 there exists a copy of P_1 where all copies of **A** with projection A_2 are monochromatic.



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• Ramsey Theorem: $\mathbf{K}_n \longrightarrow (\mathbf{K}_{|\mathbf{B}|})_2^{\mathbf{K}_{|\mathbf{A}|}}$

J. Hubička Ramsey Classes by Partite Construction I

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- Ramsey Theorem: $\mathbf{K}_n \longrightarrow (\mathbf{K}_{|\mathbf{B}|})_2^{\mathbf{K}_{|\mathbf{A}|}}$
- Construct P₀



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- Ramsey Theorem: $\mathbf{K}_n \longrightarrow (\mathbf{K}_{|\mathbf{B}|})_2^{\mathbf{K}_{|\mathbf{A}|}}$
 - $\mathbf{K}_n \longrightarrow (\mathbf{K}_{|\mathbf{B}|})_2$
- Construct P₀
- Enumerate by A₁,..., A_N all possible projections of copies of A in P₀
- Construct $\mathbf{P}_1, \dots, \mathbf{P}_N$

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- Ramsey Theorem: $\mathbf{K}_n \longrightarrow (\mathbf{K}_{|\mathbf{B}|})_2^{\mathbf{K}_{|\mathbf{A}|}}$
- Construct P₀
- Enumerate by $\mathbf{A}_1, \ldots, \mathbf{A}_N$ all possible projections of copies of A in \mathbf{P}_0
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 - **P**_i is built repeated free amalgamation of \mathbf{P}_i over all copies of \mathbf{B}_i in \mathbf{C}_i

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• Put $\mathbf{C} = \mathbf{P}_N$

Lemma

Let **A** be a structure s.t. $A = \{1, 2, ..., a\}$ and **B** be an **A**-partite system.

Then there exists a **A**-partite system **C** s.t.

 $\mathbf{C} \longrightarrow (\mathbf{B})_2^{\mathbf{A}}.$

J. Hubička Ramsey Classes by Partite Construction I

Lemma

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$$\mathbf{A} = \begin{bmatrix} 1 & X_{\mathbf{B}_{1}}^{1} \bullet & \mathbf{A}_{\mathbf{B}_{2}}^{1} \bullet \\ 2 & X_{\mathbf{B}_{1}}^{2} \bullet & \mathbf{A}_{\mathbf{A}_{2}}^{1} \bullet \\ X_{\mathbf{B}_{2}}^{2} \bullet & \mathbf{A}_{\mathbf{A}_{2}}^{2} \bullet \\ X_{\mathbf{A}_{2}}^{2} \bullet & \mathbf{A}_{\mathbf{A}_{2}}^{2} \bullet \\ X_$$

Proof by application of Hales-Jewett theorem

Theorem (Hales-Jewett theorem)

For every finite alphabet Σ there exists $N = HJ(\Sigma)$ so that for every 2-coloring of functions $h : \{1, 2, ..., N\} \to \Sigma$ there exists a monochromatic combinatorial line.

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Proof by application of Hales-Jewett theorem

Theorem (Hales-Jewett theorem)

For every finite alphabet Σ there exists $N = HJ(\Sigma)$ so that for every 2-coloring of functions $h : \{1, 2, ..., N\} \to \Sigma$ there exists a monochromatic combinatorial line.

Definition

For non-empty $\omega \subseteq \{1, 2, ..., N\}$ and $f : \{1, 2, ..., N\} \setminus \omega \to \Sigma$ combinatorial line (ω, f) is the set of all functions $f' : \{1, 2, ..., N\} \to \Sigma$ such that

$$f'(i) = egin{cases} ext{constant for } i \in \omega, \ f(i) ext{ otherwise.} \end{cases}$$

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Proof by application of Hales-Jewett theorem

J. Hubička Ramsey Classes by Partite Construction I

Proof by application of Hales-Jewett theorem

$$\mathbf{A} = \begin{bmatrix} 1 & X_{\mathbf{B}}^{1} \bullet & a \\ 2 & B = \begin{bmatrix} x_{\mathbf{B}}^{1} \bullet & x \\ y & y \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \end{bmatrix} \end{bmatrix} \begin{bmatrix} a \\ y \end{bmatrix} \end{bmatrix} \begin{bmatrix}$$

• $\Sigma = \{ {a \choose x}, {b \choose y}, {b \choose z} \}$ (alphabet describe all copies of **A** in **B**)

Proof by application of Hales-Jewett theorem

$$\mathbf{A} = \begin{bmatrix} 1 & X_{\mathbf{B}}^{1} \bullet & \mathbf{a} \\ 2 & B = \begin{bmatrix} x_{\mathbf{B}}^{1} \bullet & \mathbf{a} \\ & & & & \\ & &$$

Σ = { (^a_x), (^b_y), (^b_z) } (alphabet describe all copies of A in B)
 N = HJ(Σ)

Proof by application of Hales-Jewett theorem

$$\mathbf{A} = \begin{bmatrix} 1 & X_{\mathbf{B}}^{1} \bullet & \mathbf{a} \\ 2 & B = \begin{bmatrix} a & b \\ 0 & y & y \end{bmatrix}$$

- Σ = { (^a_x), (^b_y), (^b_z) } (alphabet describe all copies of A in B)
 N = HJ(Σ)
- Build C so that functions h: {1,2,..., N} → Σ correspond to copies of A and combinatorial lines to copies of B:

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Proof by application of Hales-Jewett theorem

$$\mathbf{A} = \begin{bmatrix} 1 & X_{\mathbf{B}}^{1} \bullet & \mathbf{a} \\ 2 & B = \begin{bmatrix} a & b \\ x_{\mathbf{B}}^{2} \bullet & \mathbf{x} & \mathbf{y} \end{bmatrix}$$

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 N = HJ(Σ)
- Build C so that functions h: {1,2,..., N} → Σ correspond to copies of A and combinatorial lines to copies of B:
 - Vertices in partition $\mathbf{X}_{\mathbf{C}}^{i}$: Functions $f : \{1, 2, \dots, N\} \rightarrow X_{\mathbf{B}}^{i}$.

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Proof by application of Hales-Jewett theorem

$$\mathbf{A} = \begin{bmatrix} 1 & X_{\mathbf{B}_{+}}^{1} \bullet & \mathbf{a} \\ 2 & B = \begin{bmatrix} a & b \\ x_{\mathbf{B}_{+}}^{2} \bullet & \mathbf{x} \end{bmatrix} \begin{bmatrix} a & b \\ y & y \end{bmatrix}$$

- Σ = { (^a_x), (^b_y), (^b_z) } (alphabet describe all copies of A in B)
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- Build C so that functions h: {1,2,..., N} → Σ correspond to copies of A and combinatorial lines to copies of B:
 - Vertices in partition $\mathbf{X}_{\mathbf{C}}^{i}$: Functions $f : \{1, 2, \dots, N\} \rightarrow X_{\mathbf{B}}^{i}$.
 - Intended embedding B → C corresponding the combinatorial line (ω, f):

$$e_{\omega,f}(v)(i)$$

 $\begin{cases} v \text{ for } i \in \omega, \\ vertex \text{ of } f(i) \text{ in the same partition as } v \text{ otherwise.} \end{cases}$

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Proof by application of Hales-Jewett theorem

$$\mathbf{A} = \begin{bmatrix} 1 & X_{\mathbf{B}_{\perp}}^{1} \bullet & \mathbf{a} \\ 2 & B = \begin{bmatrix} x_{\mathbf{B}_{\perp}}^{1} \bullet & \mathbf{a} \\ x_{\mathbf{B}_{\perp}}^{2} \bullet & \mathbf{x} \end{bmatrix} \begin{bmatrix} a & b \\ y & y \end{bmatrix}$$

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• Fact: It is possible to add tuples to relations as needed to make this work

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Easy description of C:



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Easy description of **C**:

$$\mathbf{A} = \begin{bmatrix} 1 & X_{\mathbf{B}_{1}}^{1} \bullet & \mathbf{A}_{\mathbf{B}_{2}}^{1} \bullet & \mathbf{A}_{\mathbf{B}_{2}}^{1} \bullet & \mathbf{A}_{\mathbf{B}_{2}}^{1} \bullet & \mathbf{A}_{\mathbf{A}_{2}}^{1} \bullet & \mathbf{A}_{\mathbf{A}_{2}}^{1$$

- Vertices in partition $\mathbf{X}_{\mathbf{C}}^{i}$: Functions $f : \{1, 2, \dots, N\} \rightarrow X_{\mathbf{B}}^{i}$
- Add as many tuples to relations as possible such that all the evaluation maps g_i(f) = f(i) are homomorphisms from C to B.

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Let class \mathcal{K} be a class of structures satisfying given axioms. To show that \mathcal{K} is Ramsey one can show that the partite construction preserve the axioms.

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Let class \mathcal{K} be a class of structures satisfying given axioms. To show that \mathcal{K} is Ramsey one can show that the partite construction preserve the axioms.

Nešetřil, Rödl, 1977: Classes with forbidden (amalgamation) irreducible structures

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Uses of the partite construction

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The Partite Construction





- Ramsey Theorem:
 - $K_n \longrightarrow (\mathbf{K}_{|\mathbf{B}|})_2^{\mathbf{K}_{|\mathbf{A}|}}$
- Construct P₀
- Enumerate by A₁,..., A_N all possible projections of copies of A in P₀
- Construct **P**₁,...,**P**_N
 - B_i: partite system induced on P_{i-1} by all copies of all with projection to A_i
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• Put $\mathbf{C} = \mathbf{P}_N$

The Induced Partite Construction





- Nešetřil-Rödl Theorem: $\textbf{C_0} \longrightarrow (\textbf{B})_2^{\textbf{A}}$
- Construct C₀-partite P₀
- Enumerate by A₁,..., A_N all possible projections of copies of A in P₀
- Construct C_0 -partite P_1, \ldots, P_N :
 - B_i: partite system induced on P_{i-1} by all copies of all with projection to A_i
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• Put $C = P_N$ If K is irreducible and A, B are K-free, then so is C.

An exotic example



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Structure of bow-tie-free graphs



Edges in no triangles

Edges in 1 triangle

Edges in 2+ triangles

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Structure of bow-tie-free graphs



Edges in no triangles Edges in 1 triangle Edges in 2+ triangles

Definition

Chimney is a graph created by gluing multiple triangles over one edge.





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Graph is good if every vertex is either in a chimney or K_4 .

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Graph is good if every vertex is either in a chimney or K_4 .

• Every bowtie-free graph G is a subgraph of some good graph G'.

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Graph is good if every vertex is either in a chimney or K_4 .

- Every bowtie-free graph *G* is a subgraph of some good graph *G*'.
- For every good graph G = (V, E) the graph G_Δ = (V, E_Δ) (E_Δ are edges in triangles) is a disjoint union of copies of chimneys and K₄.

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- For every good graph G = (V, E) the graph G_Δ = (V, E_Δ) (E_Δ are edges in triangles) is a disjoint union of copies of chimneys and K₄.

Closure of a vertex v = all endpoints of red edges contained in triangles containing v.

Lemma

Bow-tie-free graphs have free amalgamation over closed structures.

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Ramsey property of bow-tie free graphs

3 types of vertices and their closures:



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Ramsey property of bow-tie free graphs

3 types of vertices and their closures:



To describe lift of bowtie graphs we only need to forbid all triangles except for B-B-R and R-R-R.

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Ramsey property of bow-tie free graphs

3 types of vertices and their closures:



To describe lift of bowtie graphs we only need to forbid all triangles except for B-B-R and R-R-R.

Theorem (H., Nešetřil, 2014)

The class of graphs not containing bow-tie as non-induced subgraph have Ramsey lift.

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Unary closure description C is a set of pairs (R^U, R^B) where R^U is unary relation and R^B is binary relation.

We say that structure **A** is C-closed if for every pair (R^U, R^B) the B-outdegree of every vertex of **A** that is in U is 1.

Theorem (H., Nešetřil, 2015)

Let \mathcal{E} be a family of complete ordered structures and \mathcal{U} an unary closure description. Then the class of all C-closed structures in Forb_E(\mathcal{E}) has Ramsey lift.

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All Cherlin Shelah Shi classes with unary closure can be described this way!

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- Nešetřil-Rödl Theorem: $C_0 \longrightarrow (B)_2^A$
- Construct C₀-partite **P**₀

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- Nešetřil-Rödl Theorem: $C_0 \longrightarrow (B)_2^A$
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• Put $\mathbf{C} = \mathbf{P}_N$





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• Put $\mathbf{C} = \mathbf{P}_N$

A, **B**, **B**_{*i*} are C-closed. Only potential problem is the partite construction.

Easy description of C:

$$\mathbf{A} = \begin{bmatrix} X_{\mathbf{B}}^{1} \bullet \mathbf{A}^{a} \\ B = \begin{bmatrix} X_{\mathbf{B}}^{2} \bullet \mathbf{A}^{a} \\ X_{\mathbf{B}}^{2} \bullet \mathbf{A}^{c} \bullet \mathbf{A} \end{bmatrix} \begin{bmatrix} x_{\mathbf{B}}^{b} \bullet \mathbf{A}^{c} \\ \mathbf{A}^{c} \bullet \mathbf{A}^{c} \bullet \mathbf{A}^{c} \bullet \mathbf{A}^{c} \end{bmatrix}$$

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Easy description of C:

$$\mathbf{A} = \begin{bmatrix} X_{\mathbf{B}}^{1} \bullet \mathbf{A}^{a} \\ \mathbf{B} = \begin{bmatrix} X_{\mathbf{B}}^{2} \bullet \mathbf{A}^{a} \\ X_{\mathbf{B}}^{2} \bullet \mathbf{A}^{a} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{a} \bullet \mathbf{A}^{b} \\ \mathbf{A}^{a} \bullet \mathbf{A}^{b} \end{bmatrix}$$

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J. Hubička Ramsey Classes by Partite Construction I

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J. Hubička Ramsey Classes by Partite Construction I

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- Nešetřil-Rödl Theorem:
 C₀ → (B)^A₂.
- Construct *C*₀-partite **P**₀



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- Nešetřil-Rödl Theorem: $C_0 \longrightarrow (B)_2^A$.
- Construct C₀-partite **P**₀
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 - unify vertices to preserve out-degree 1 of non-unary closure edges

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Denote by $\mathbb{Q}\mathbb{Q}$ the structure with binary relation \leq and ternary relation \prec with the following properties

- relation \leq forms the generic linear order
- If or every vertex a ∈ QQ the relation {(b, c) : (a, b, c) ∈ ⊰} forms the generic linear oder on QQ \ {a} that is free to ≤.

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Use alternative representation with binary relations and closures to show that the age of $\mathbb{Q}\mathbb{Q}$ is a Ramsey class:

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- If or every vertex a ∈ QQ the relation {(b, c) : (a, b, c) ∈ ⊰} forms the generic linear oder on QQ \ {a} that is free to ≤.

Use alternative representation with binary relations and closures to show that the age of $\mathbb{Q}\mathbb{Q}$ is a Ramsey class:



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The End



J. Hubička Ramsey Classes by Partite Construction I

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